Linear partial differential equation

Formation :→ If Z is a function of two independent variables x; y. Then we have a differential equation of the form $f(x, y, z, p, q) = 0$ which is called the partial differential equation of 1st order.

Where $p = \frac{\partial z}{\partial x}$, & $q = \frac{\partial z}{\partial y}$

(A) Formation by the elimination of arbitrary const :→

Let us we have a function

$$f(x, y, z, a, b) = 0 \quad \text{— (1)}$$

Differenceating w.r.to x, we get

$$\therefore \frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial x} = 0$$

$$\therefore \frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} p = 0 \quad \text{— (1)}$$

Again differenceating w.r.to y, we get

$$\frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} q = 0 \quad \text{— (2)}$$

Eliminating a, b, from eq^n (1) ~~&~~, (2) & (3) we get the partial differential equation of the form $\phi(x, y, z, P, q) = 0$

(B) Formation by the elimination of arbitrary function :→

→ Let u, v be two function of x, y, z connected by the relation

$$f(u, v) = 0 \quad \text{— (1)}$$

Differenceating (1) w.r.to x & y we get

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot p\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}p\right] = 0 \quad \text{— (2)}$$

$$\& \ \frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}q\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}q\right] = 0 \quad \text{— (3)}$$

Eliminating $\frac{\partial f}{\partial u}$ & $\frac{\partial f}{\partial v}$ from equation (2) & (3), we get.

$$\begin{vmatrix} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}p & \frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}p \\ \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}q & \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}q \end{vmatrix} = 0$$

$$\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}p\right)\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}q\right) - \left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}q\right)\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}q\right) = 0$$

r, $p\left(\frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial z}\right) + q\left(\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial z} - \frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial x}\right)$

$$+ \left(\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial x}\right) = 0$$

, $p\left(\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial z} - \frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial y}\right) + q\left(\frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial z}\right)$

$$= \frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial u}{\partial x}$$

or, $P\left(\frac{\partial(u,v)}{\partial(y,z)}\right)+q\left(\frac{\partial(u,v)}{\partial(z,x)}\right)=\frac{\partial(u,v)}{\partial(x,y)}$

If we denote $P=\frac{\partial(u,v)}{\partial(y,z)}$, $Q=\frac{\partial(u,v)}{\partial(z,x)}$

$$R=\frac{\partial(u,v)}{\partial(x,y)}$$

Then it is

$Pp+Qq=R$, is the

required partial differential eqn of order one [This is the standered form)

Formation using (A):- Let $z=(x+a)(y+b)$ —①
Differenceating w.r.to x & y
we get
$p=(y+b)$ —②
& $q=(x+a)$ —③
eliminating a, b, from eqn ①, ②, ③ we get the required differential equation

$\therefore\ z=p.q$

99(A)
Formation using B:-

Let $f(x+y+z,\ x^2+y^2-z^2)=0$ —①

Let us denote $u=u(x,y,z)=x+y+z$

& $v=v(x,y,z)=x^2+y^2-z^2$

We obtain eqn ① becomes $f(u,v)=0$ —②

Differenceating (2) w.r.to x & y, we get

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}p\right]+\frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial x}+\frac{\partial v}{\partial z}p\right]$$

$$=\frac{\partial f}{\partial u}\left[1+1\cdot p\right]+\frac{\partial f}{\partial v}\left[2x-2zp\right]=0$$

or, $$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}q\right]+\frac{\partial v}{\partial y}\left[\frac{\partial v}{\partial y}+\frac{\partial v}{\partial z}\cdot q\right]$$

$$=\frac{\partial f}{\partial u}\left[1+1\cdot q\right]+\frac{\partial f}{\partial v}\left[2y-2zq\right]=0$$

eliminating $\frac{\partial f}{\partial u}$ & $\frac{\partial f}{\partial v}$ we get

$$\begin{vmatrix} 1+p & 2(x-pz) \\ 1+q & 2(y-qz) \end{vmatrix}=0$$

or $2(1+p)(y-qz)-2(1+q)(x-pz)=0$

or, $y-qz+py-p\cancel{q}z-x+pz-qx+p\cancel{q}z=0$

or, $p(y+z)+q(-z-x)=x-y$

i.e $Pp+Qq=R$ from

which is required partial differential equation.